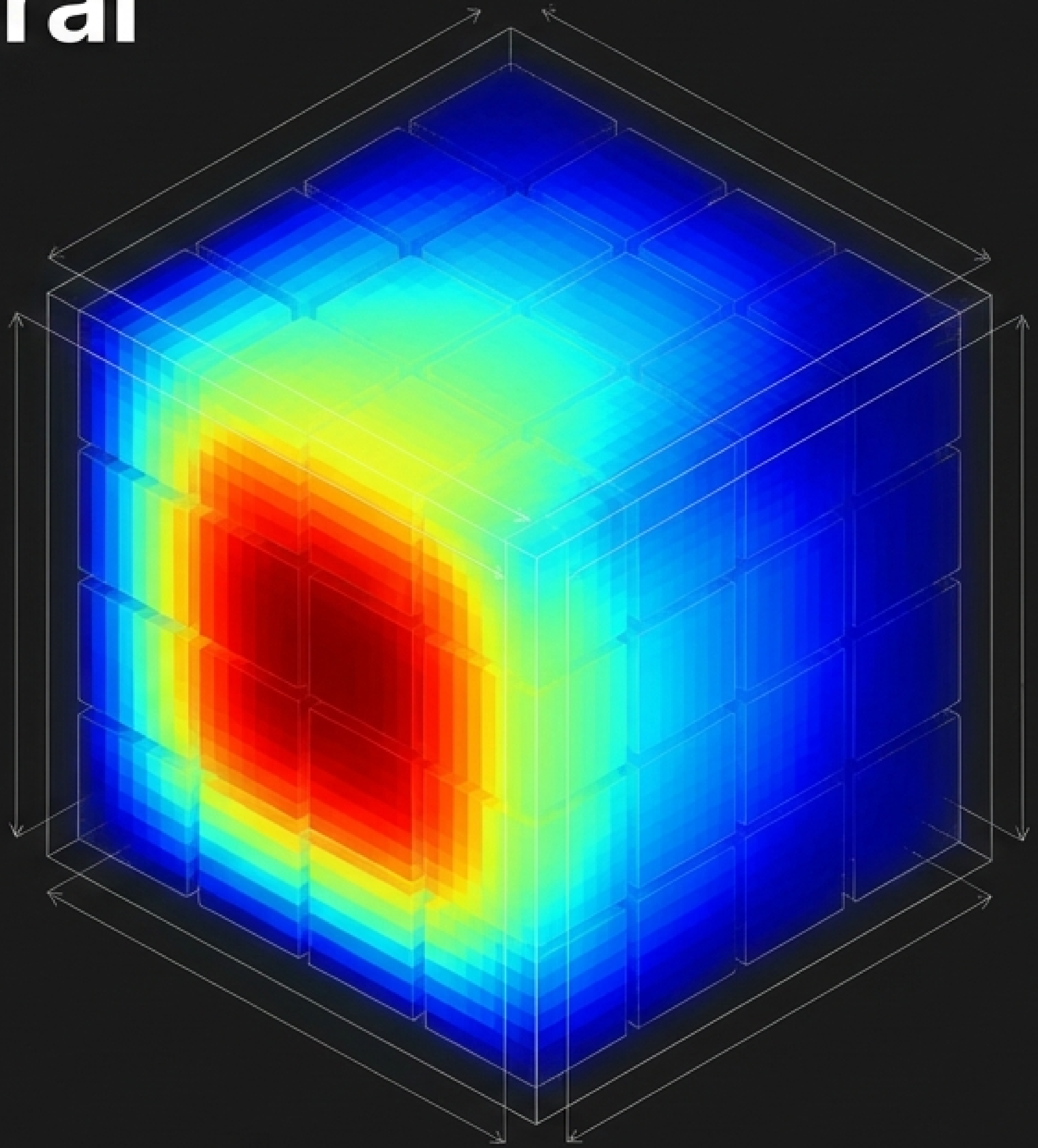
Physics-Informed Neural Networks for Fast 3D Consolidation

A Surrogate Modelling Application
for Real-Time Geotechnical Prediction



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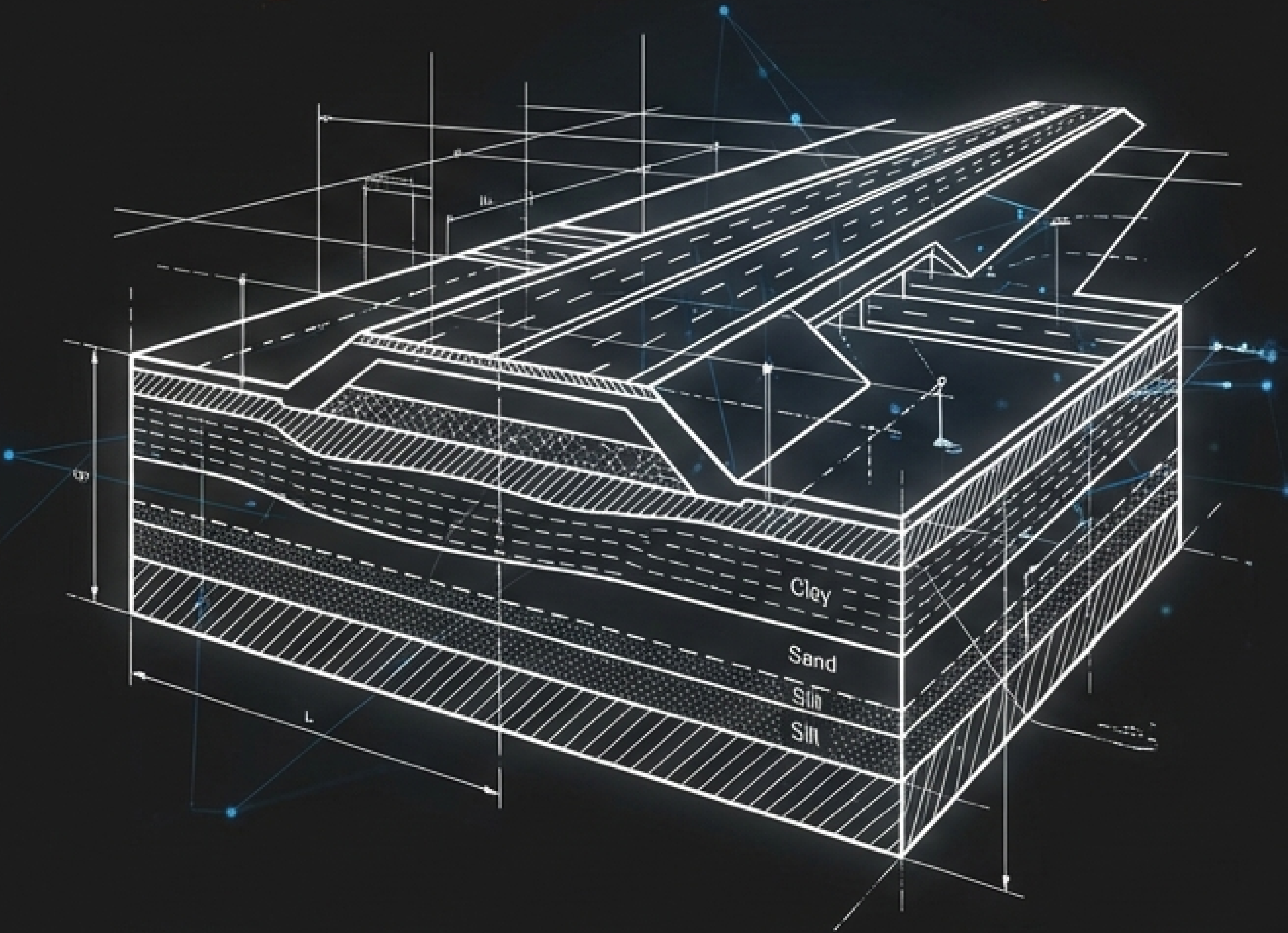
The Foundation

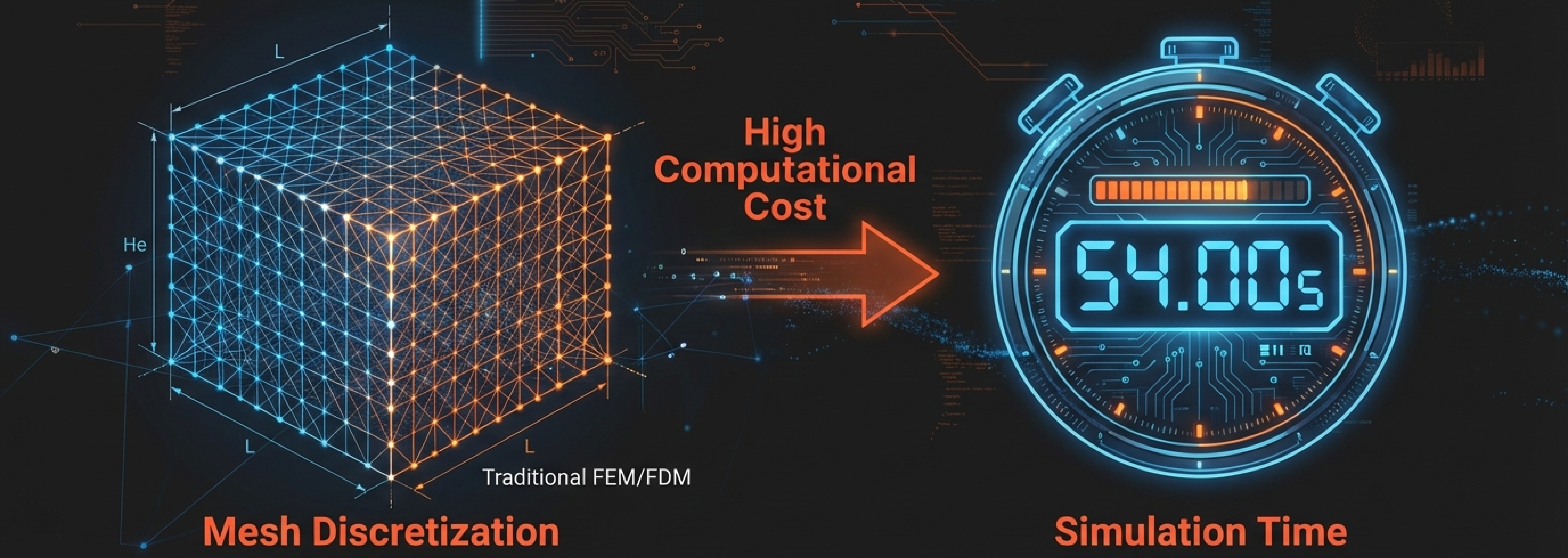


The Physics

$$\frac{\partial u}{\partial t} = c_v \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

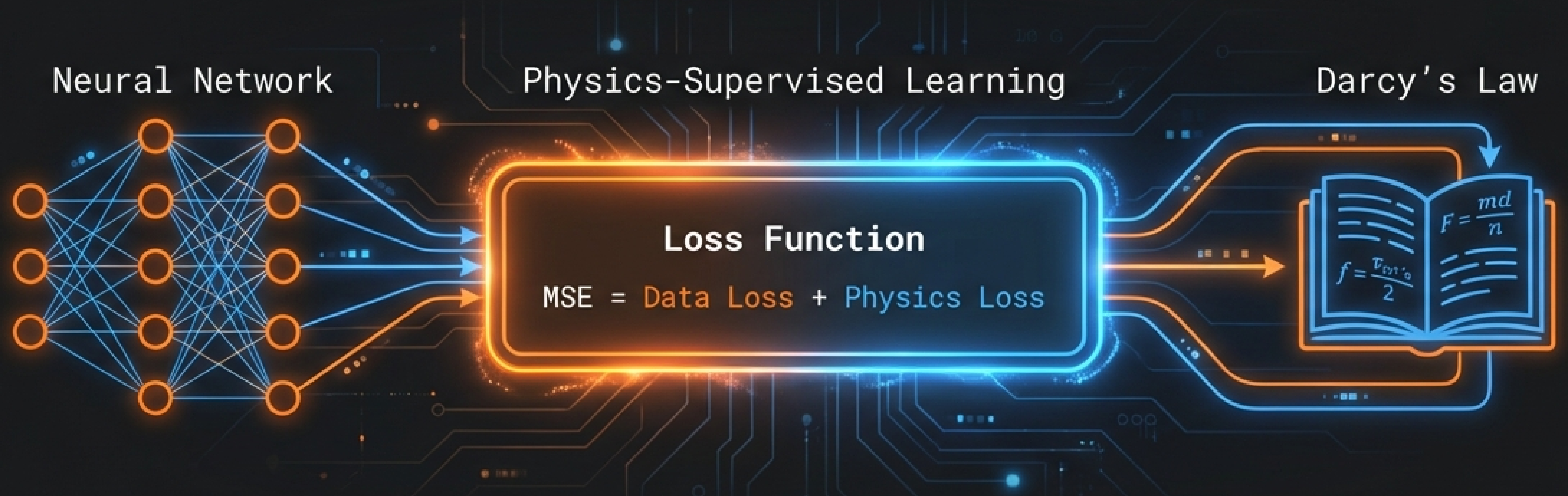
Predicting the dissipation of excess pore water pressure (u) in complex 3D environments is critical to preventing structural failure.

The Computational Bottleneck



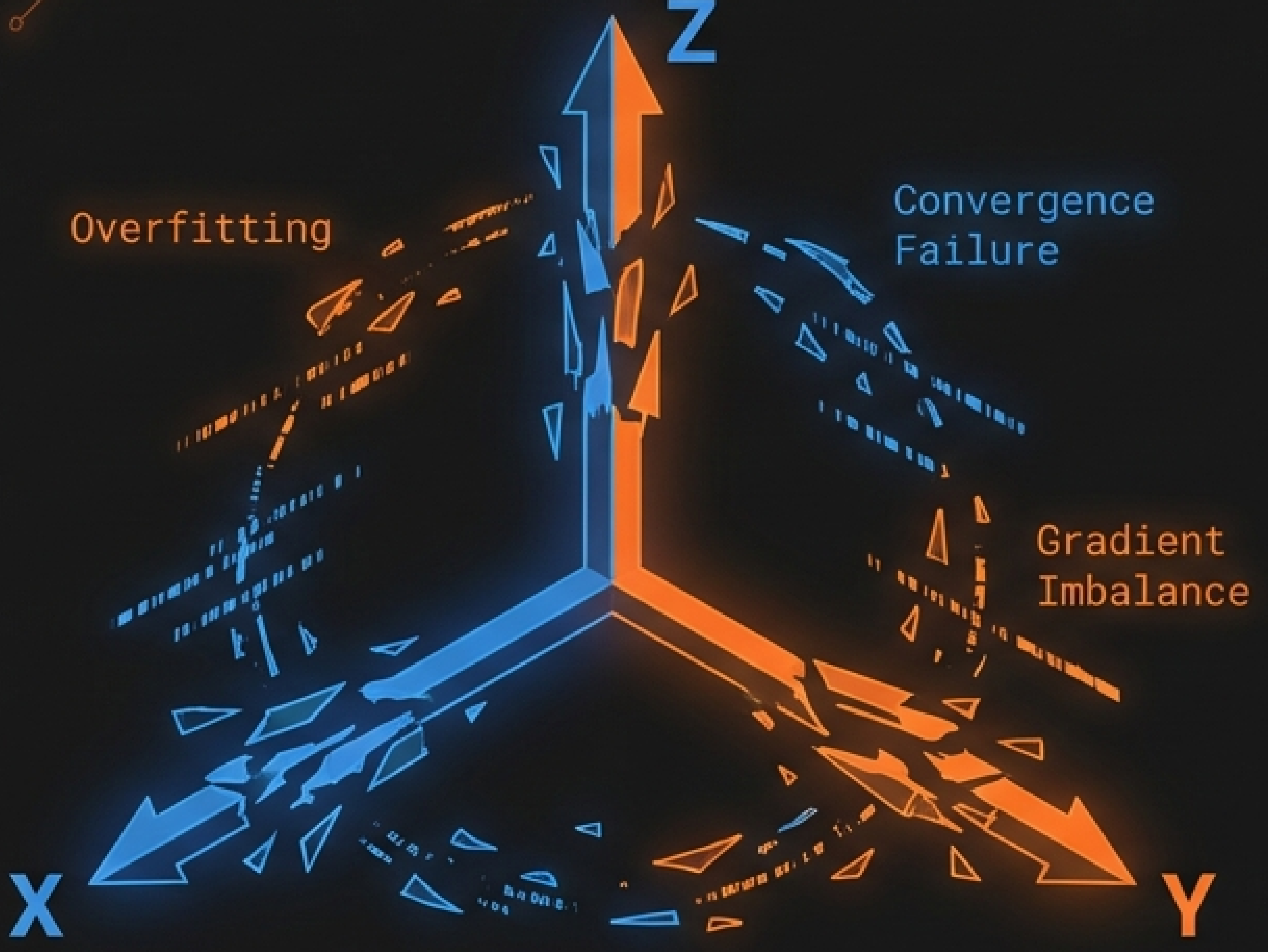
1. Iterative solvers struggle with Inverse Problems (finding parameters from data).
2. Mesh generation is computationally expensive.
3. ~54 seconds per simulation is too slow for real-time monitoring.

Encoding Laws into Logic



- Physics-Informed Neural Networks (PINNs) embed physical laws directly into the loss function.
- No massive labeled datasets required. The physics is the supervisor.
- Solves both Forward (Prediction) and Inverse (Parameter ID) problems.

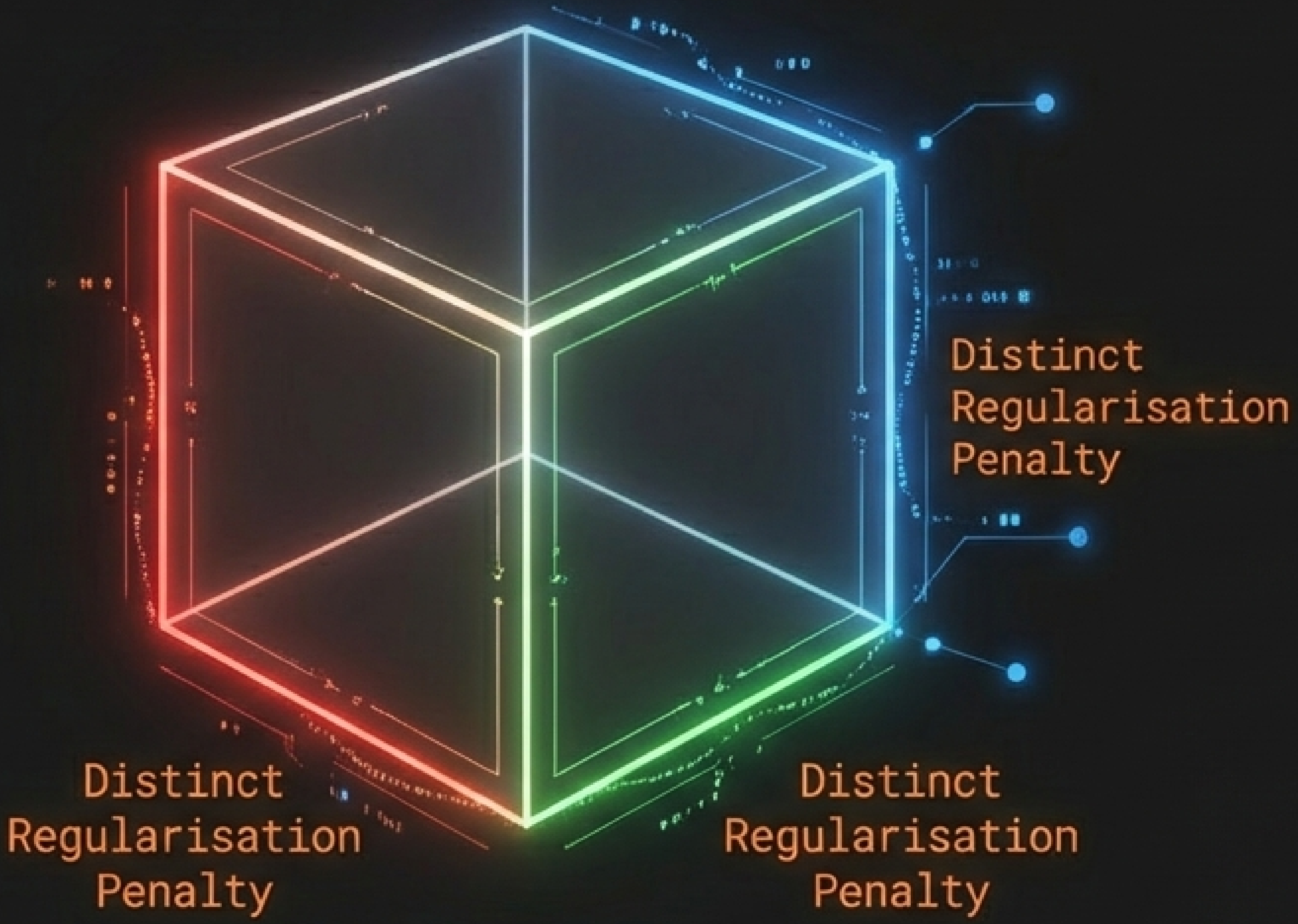
The 3D Challenge



The Curse of Dimensionality

- Extending 1D Terzaghi theory to 3D introduces exponential spatial complexity.
- Standard PINNs suffer from convergence issues due to inconsistent loss functions across x, y, and z dimensions.

Innovation I: Explicit Directional Framework



We explicitly account for directional variations in space.

Mechanism: Specific weights for x, y, and z axes ensure uniform convergence.

Result: Captures anisotropic characteristics of 3D soil behavior.

Innovation II: Two-Stage Training Strategy

Stage 1: Importance Resampling

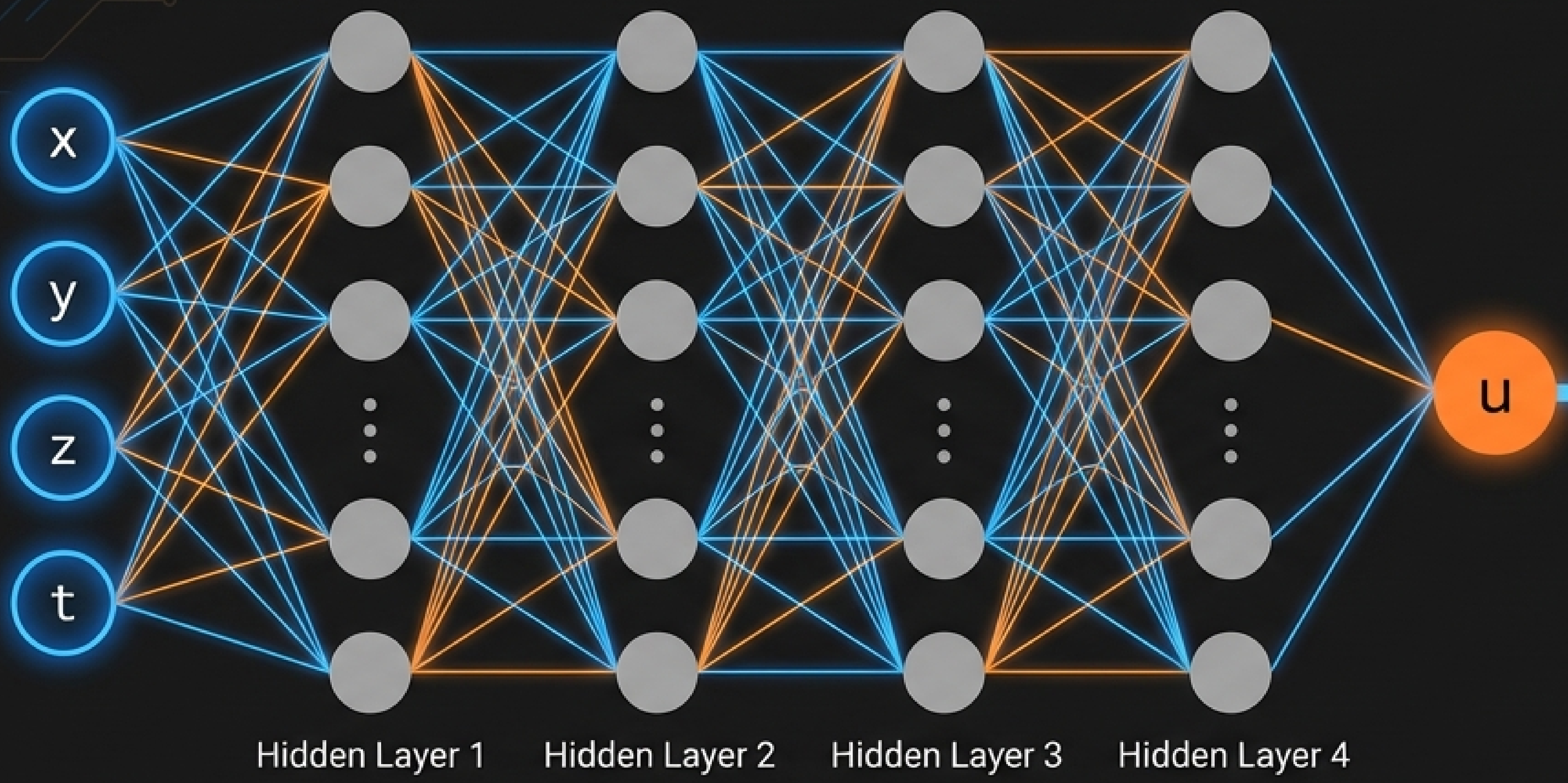
Detects high-residual "hard" areas and resamples points there.

Stage 2: Adaptive Weights (LBA)

Dynamically adjusts loss weights so no single constraint dominates.

Prevents the model from being lazy and forces accurate boundary learning.

The Architecture



$$L_f = \frac{1}{N_f}\sum \left|\frac{\partial u}{\partial t} - \nabla\cdot(D\nabla u) - f\right|^2$$

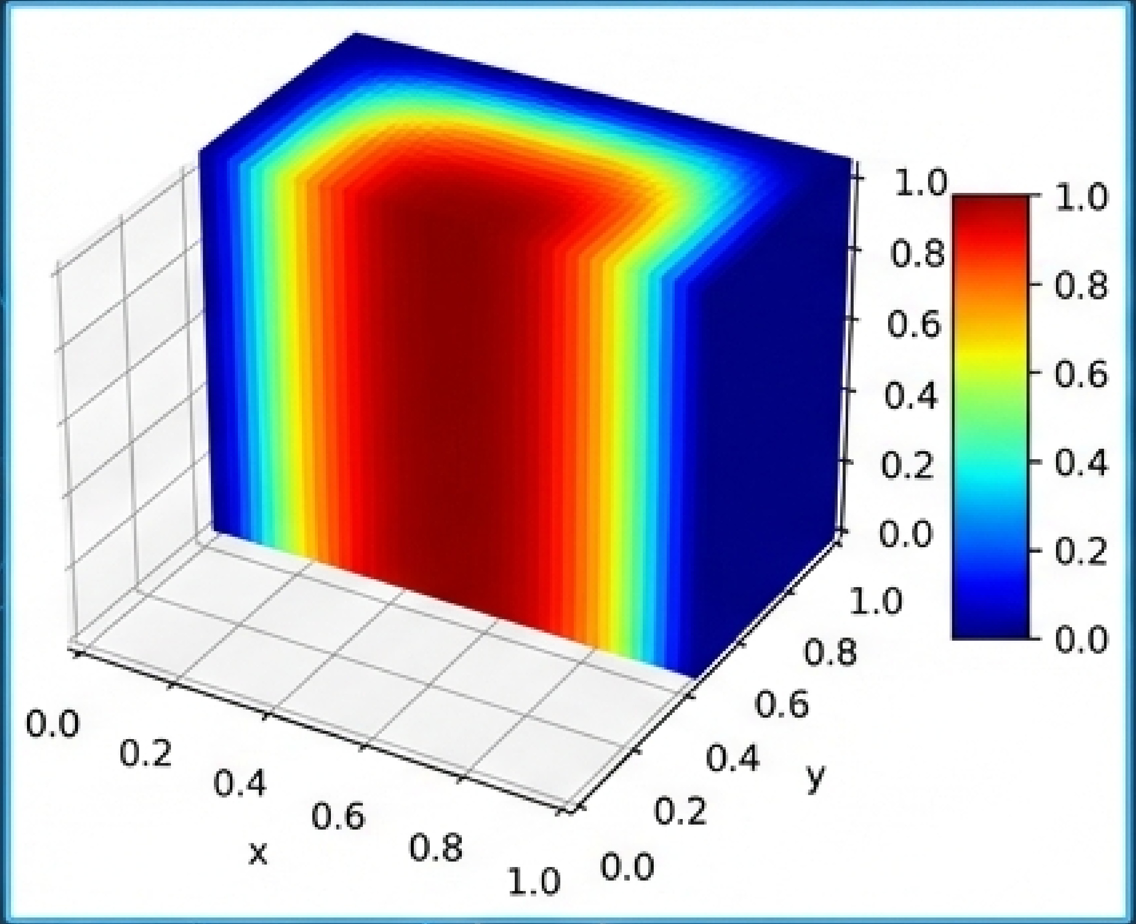
$$L_b = \frac{1}{N_b}\sum |u - u_b|^2$$

$$L_0 = \frac{1}{N_0}\sum |u - u_0|^2$$

Structure: Fully Connected Neural Network (FCNN)
Depth: 4 Hidden Layers
Width: 40 Neurons per layer (Optuna Optimized)
Activation: Tanh (Hyperbolic Tangent)

Proof of Concept: Forward Analysis

Ground Truth (Numerical)



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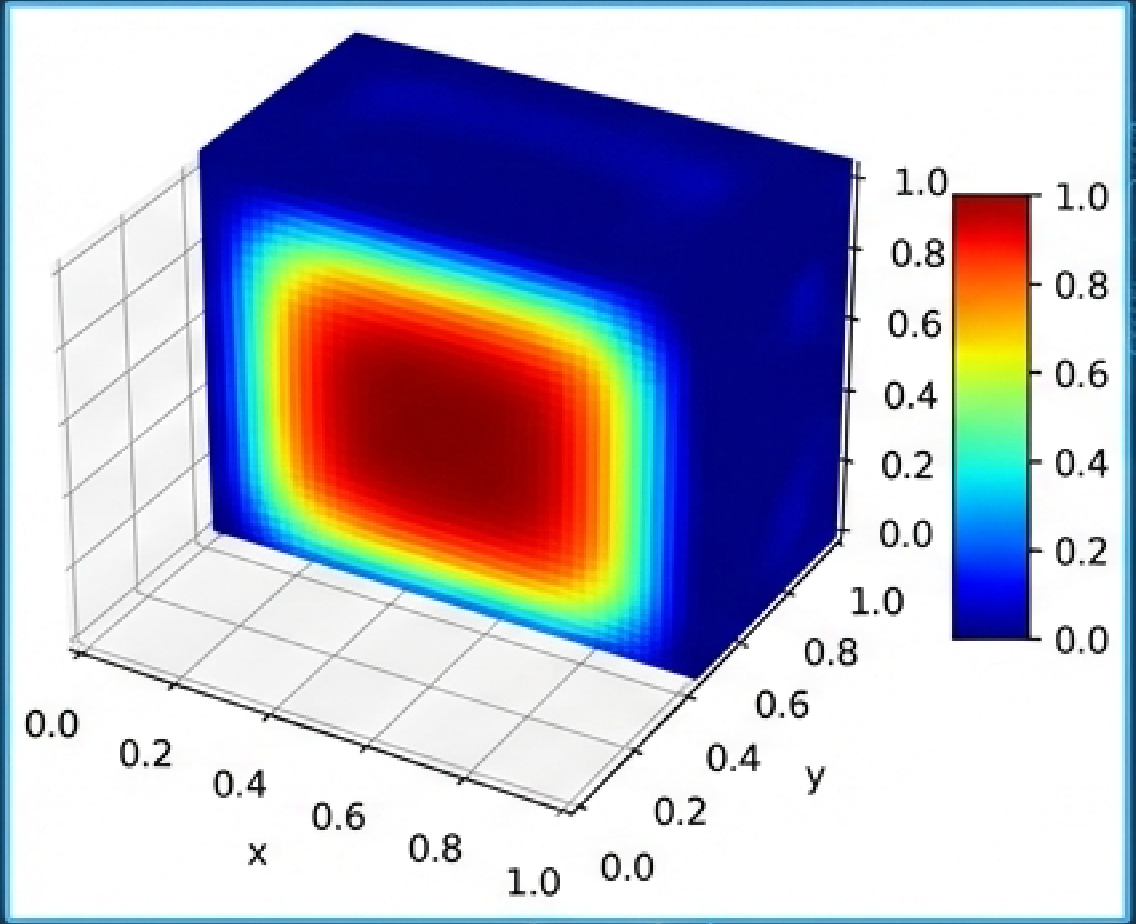
PINN Prediction

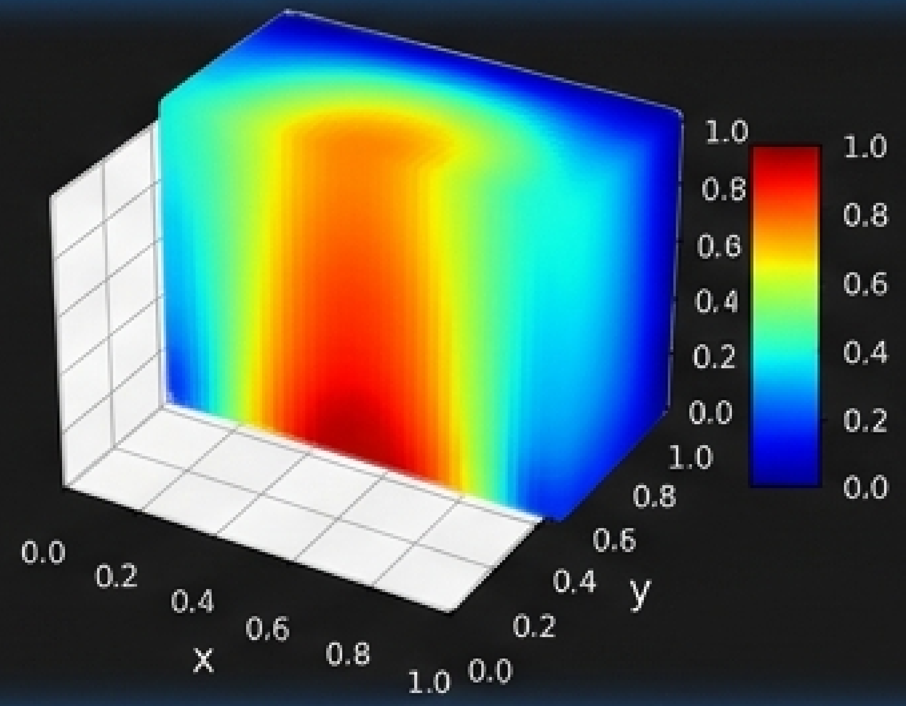


Accuracy: > 98%

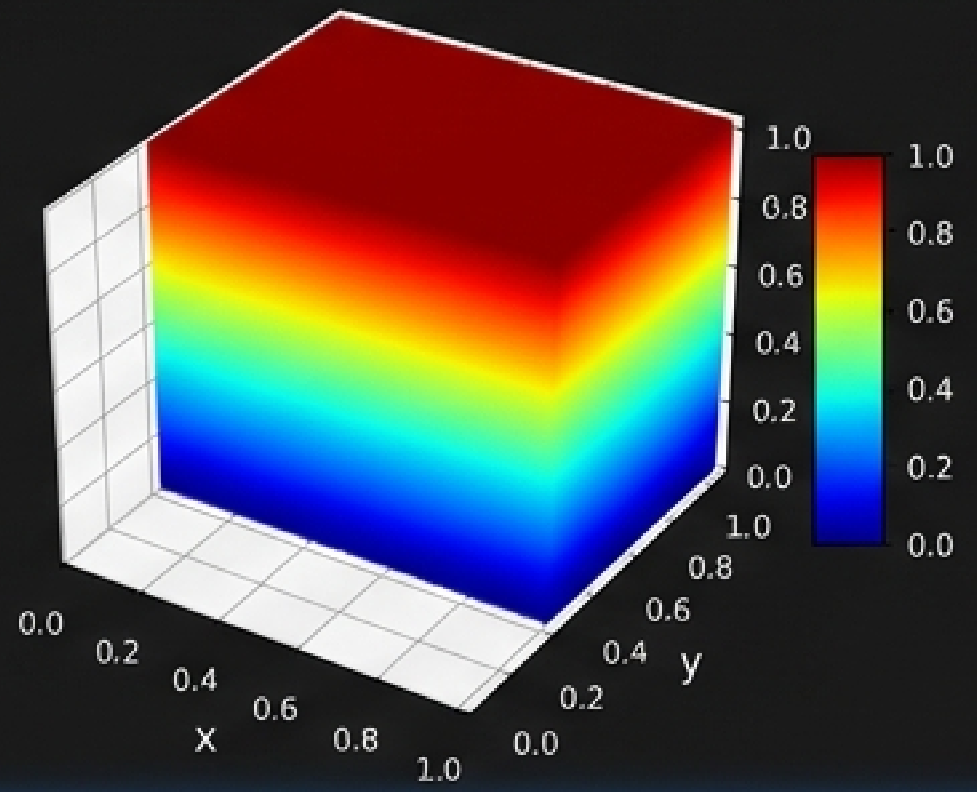
MSE Loss: ~ 10^-6

Handling Real-World Complexity

Variable Surface Load

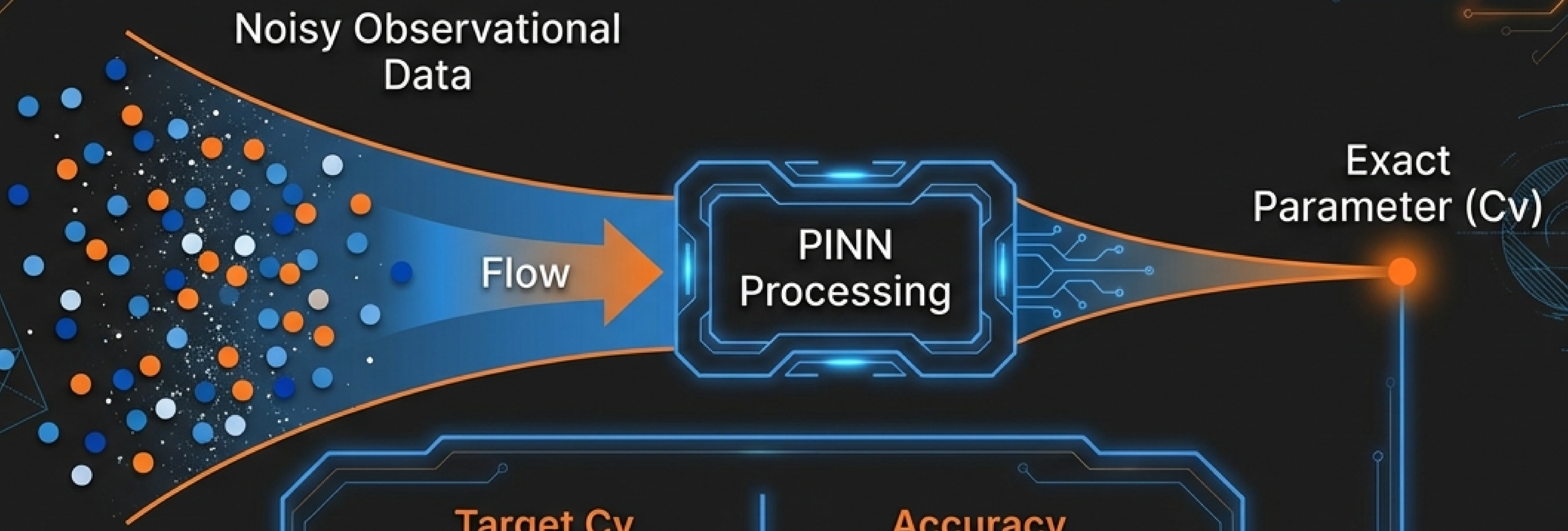


Heterogeneous Layers (Cv varies with z)



1. Models depth-dependent consolidation coefficients.
2. Handles spatially non-uniform parabolic loads.
3. Maintains ~98.8% accuracy in complex geological conditions.

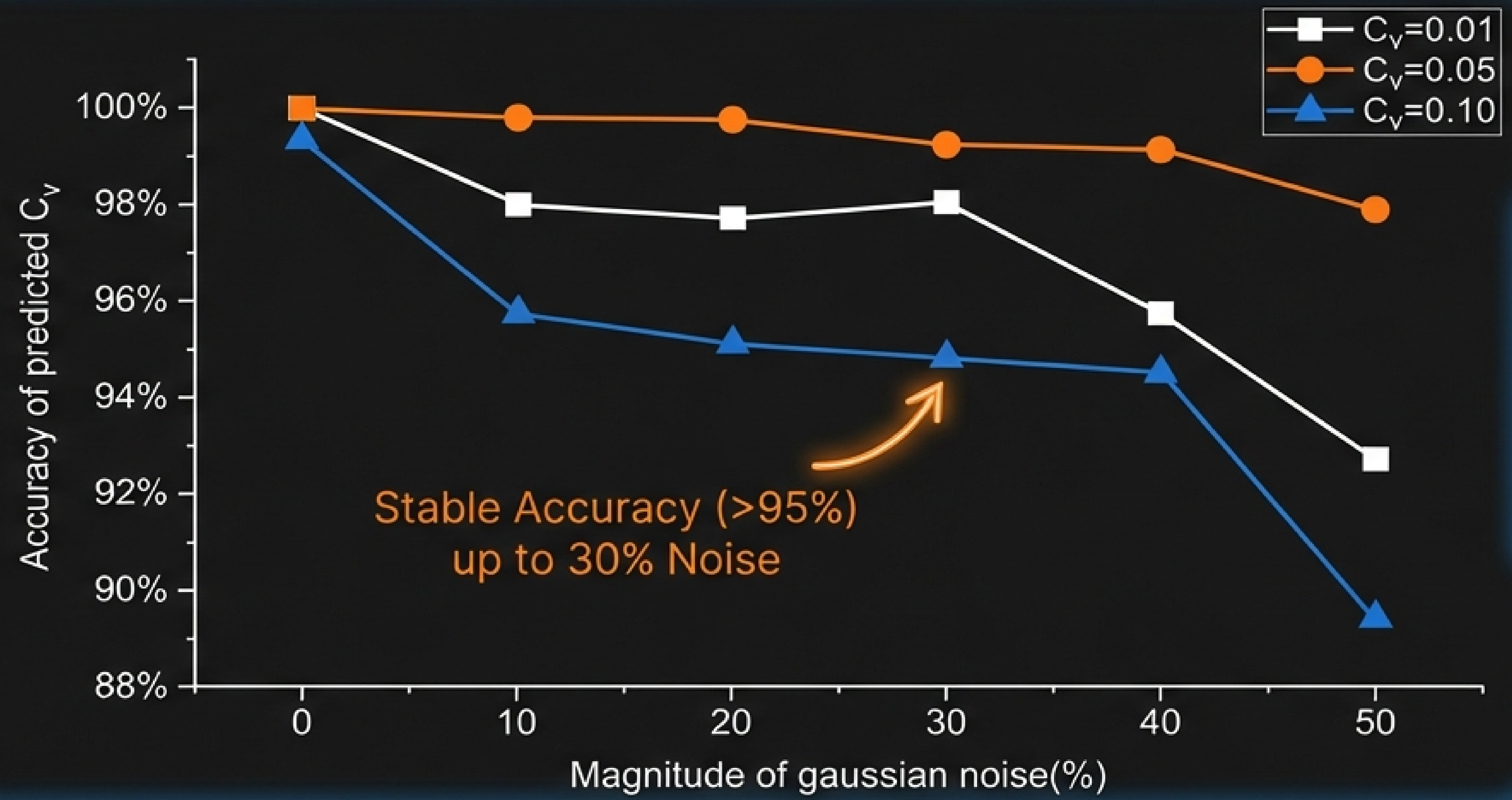
The Inverse Power: Parameter Identification



Target Cv	Accuracy
Target Cv = 0.01	Accuracy: **99.99%**
Target Cv = 0.05	Accuracy: **99.80%**
Target Cv = 0.10	Accuracy: **99.31%**

Identifies soil properties <1% error using only sparse data.

Robustness Against Noise



Physics constraints act as a filter, rejecting non-physical noise from the training data.

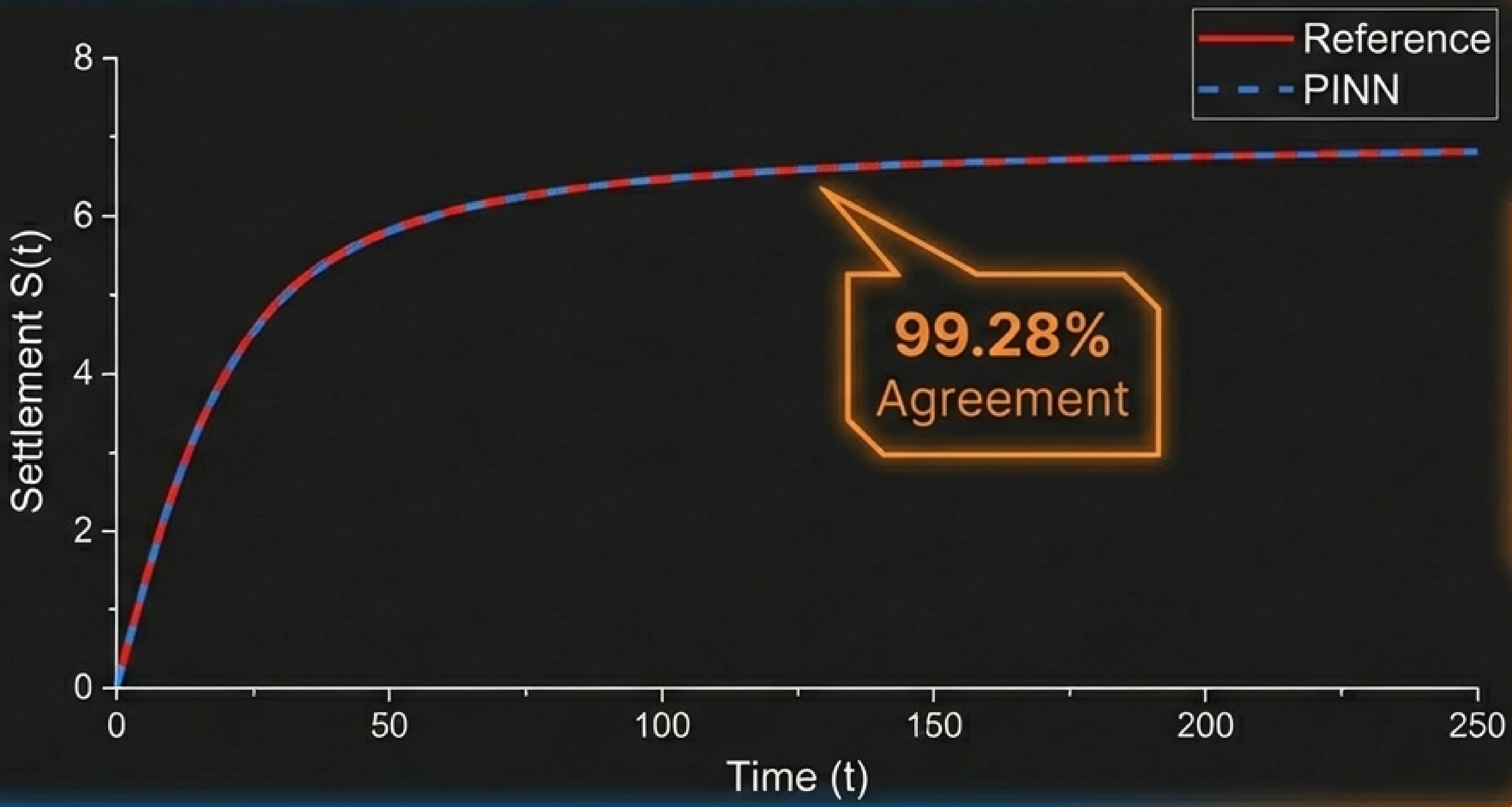
The Speed of a Surrogate

Numerical Method: 54.00s

PINN Surrogate: 0.52s

Instantaneous prediction enables real-time geotechnical monitoring systems.

Predicting Settlement S(t)



Translates pressure dissipation into actionable settlement predictions over time.

A New Era for Geotechnical Monitoring

1. Explicit 3D Directional Framework
2. Real-Time Surrogate Speed (<1s)
3. Robust Inverse Parameter ID

PINNs bridge the gap between physical theory and real-time intelligent infrastructure.