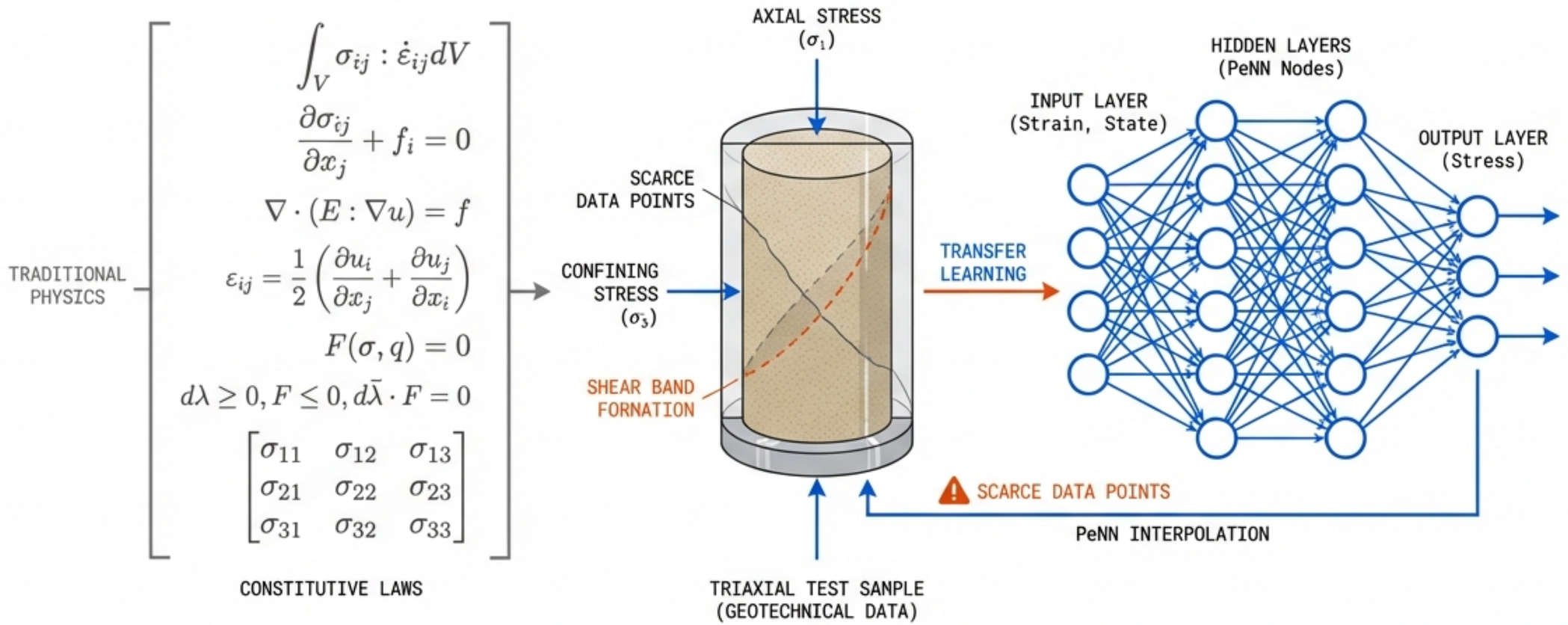
Sparse Data-Driven ConstitutiveModeling via Transfer Learning

Integrating Physics-Encoded Neural Networks (PeNNs) with Finite Element Method (FEM) simulations to solve geotechnical data scarcity.



Based on research by Zhihui Wang & Roberto Cudmani, Technical University of Munich.

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The Constitutive Modeling Dilemma in Geotechnics

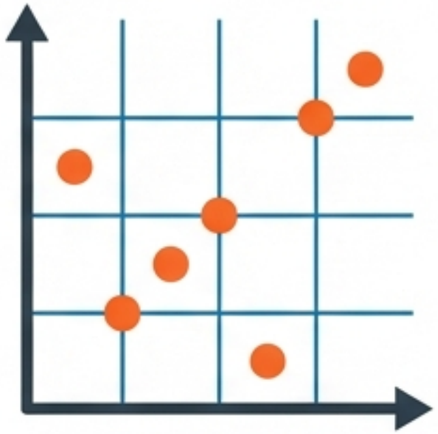
Traditional Models

- Physically consistent (Thermodynamics, Objectivity).
- Rigid structure.
- Difficult calibration for specific sands.

Pure AI Models

- High flexibility.
- Data hungry (needs 1000s of points).
- Often violates physical laws.
- Black box nature.

The Engineering Reality



- Projects provide sparse data (5–10 tests).
- Experimental noise is high.
- Safety is non-negotiable.

The Goal Enforce **Hard Physics Constraints** while Learning from **Sparse Reality**

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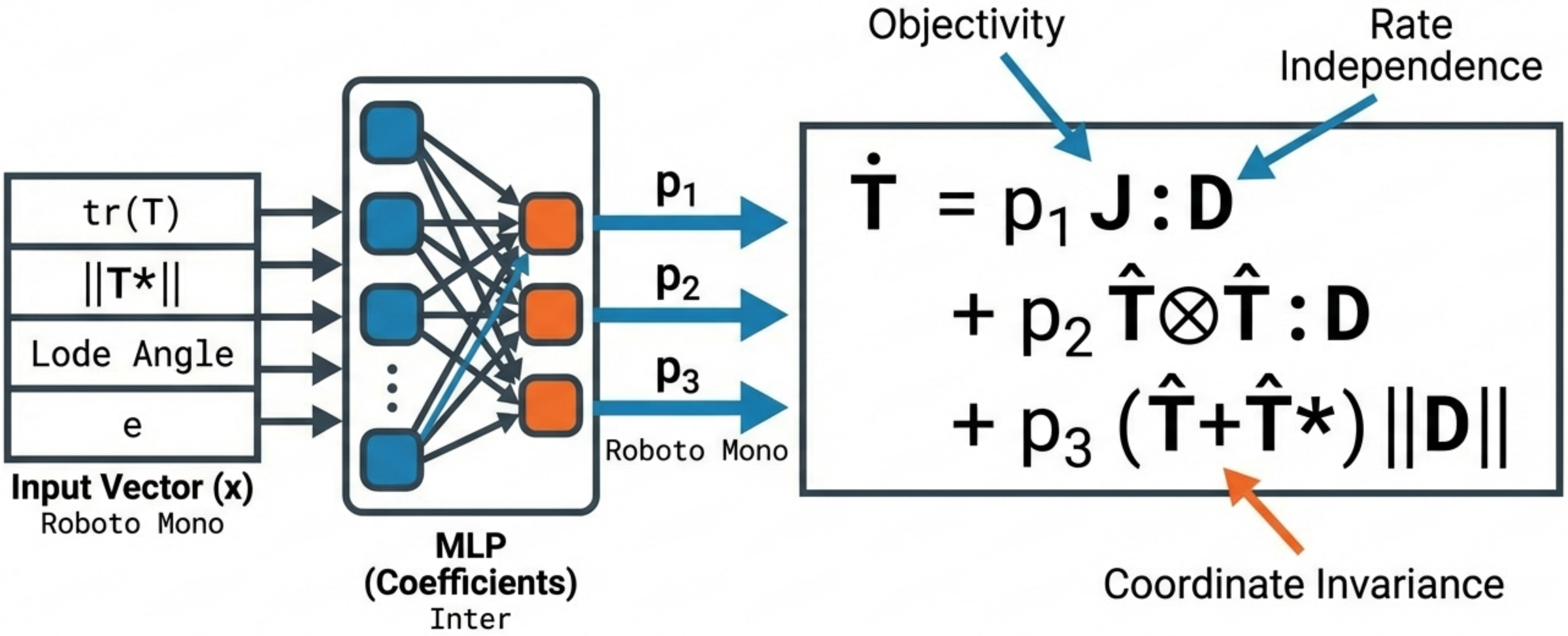
Architecture: Enforcing Hard Physical Constraints



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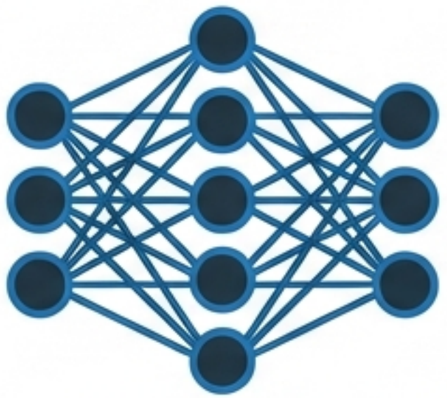
The Strategy: Transfer Learning from Synthetic to Real

Phase 1: Pre-training (Low-Fidelity)

Synthetic Data
(Hypoplasticity)

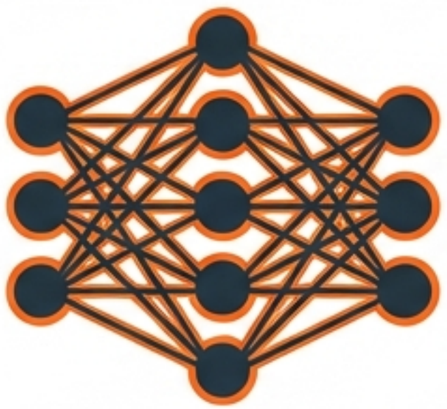


Learn General
Physics

Roboto Mono: Abundant data, covers all stress states. Physics-compliant foundation.

Phase 2: Fine-tuning (High-Fidelity)

Sparse Experimental Data
(5-10 Tests)



Learn General
Physics

Roboto Mono: Scarce, noisy data. Calibrates to specific material reality.

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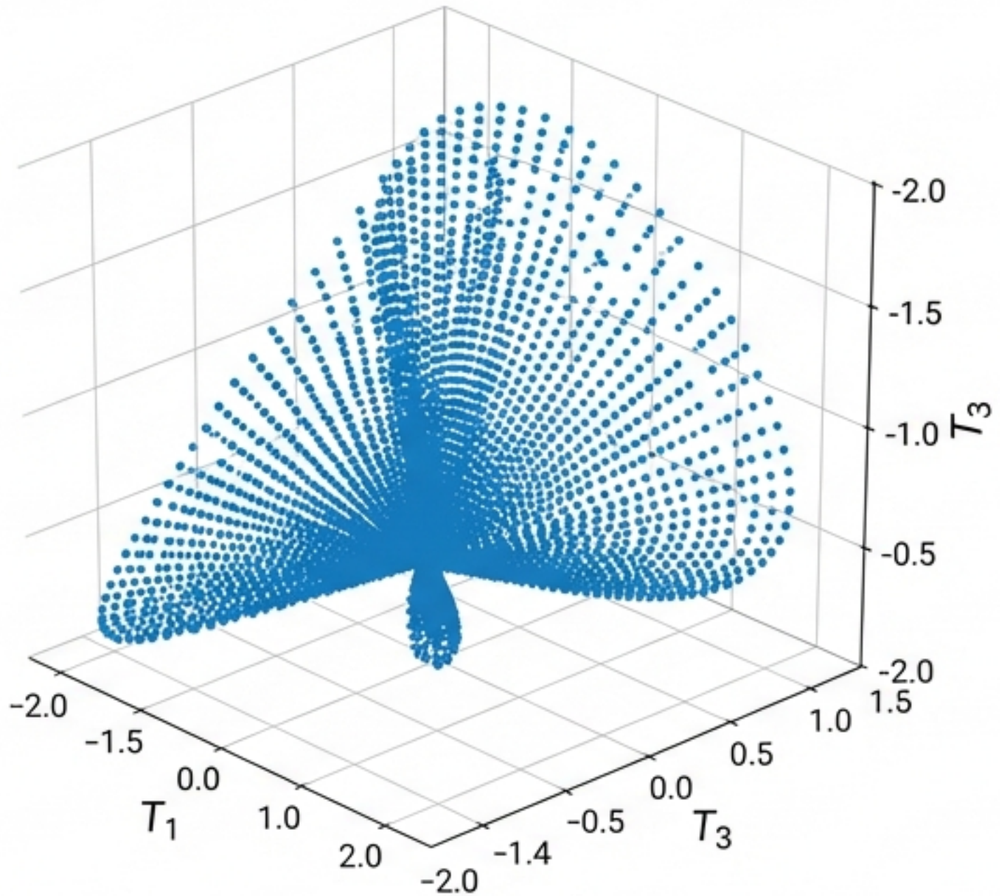
Phase 1: Pre-training with Synthetic Knowledge

Specifications

- **Source**: Reference Hypoplastic Model (Matsuoka-Nakai failure criterion).
- **Target:** Learn mapping of invariants to coefficients p_1, p_2, p_3.
- **Accuracy:** $R^2 \approx 0.9999$ (Perfect mimicry).



Sampling Strategy in Principal Stress Space

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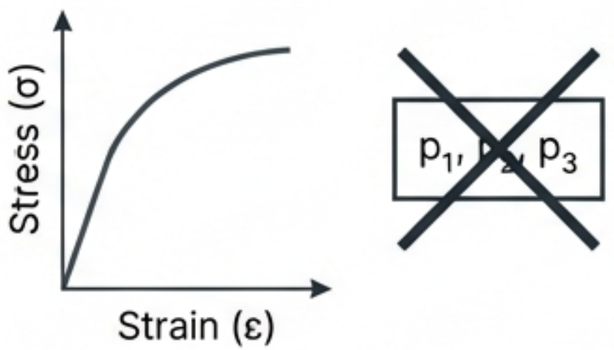
Phase 2: Fine-tuning with Implicit Experimental Data

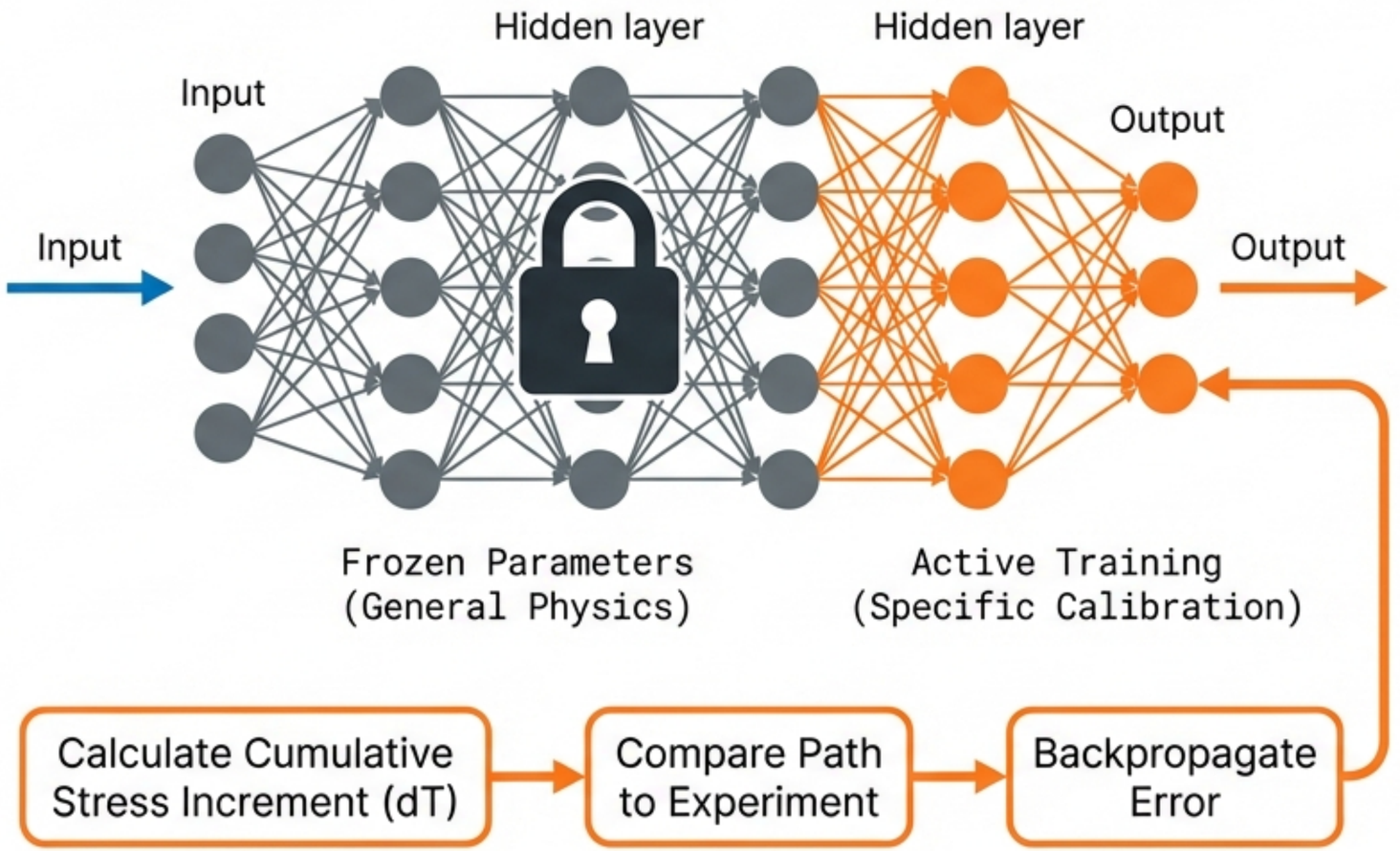
The Challenge

Inter: Experiments Experiments provide Stress/Strain curves, not coefficients p_1, p_2, p_3.

Cannot use direct supervised learning.



The Solution



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Navigating Noise with Robust Loss Functions

Primary Loss (E_r)
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$$E_r = \sum |d\tilde{T} - dT| \,/\, 9N$$

Stress Increment Error. Minimizes deviation in the stress path step-by-step.

Augmented Loss (E_a)
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$$E_a = E_{mag} + E_{dir}$$

Physics Constraints.
- Magnitude: Matches pressure and deviatoric stress.
- Direction: Maximizes cosine similarity between predicted and real stress vectors.

Why Augmented Loss? It prevents the model from overfitting to experimental noise and stabilizes convergence in data-scarce regimes.

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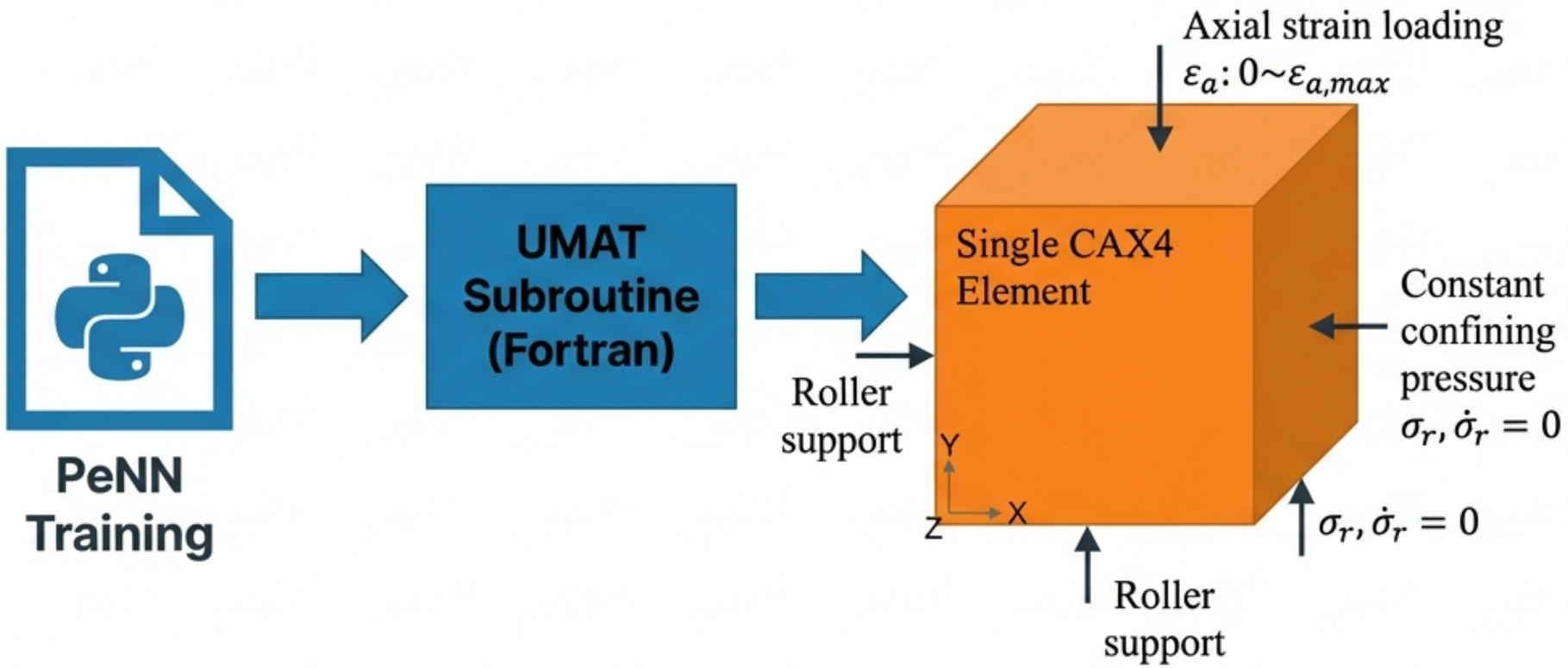
Integration into Finite Element Method (FEM)



The Acid Test: **Material-point accuracy (low training error) ≠ Simulation stability.**

Errors accumulate **step-by-step** in FEM. Validation requires full Boundary Value Problem (BVP) simulation in Abaqus/ Standard.

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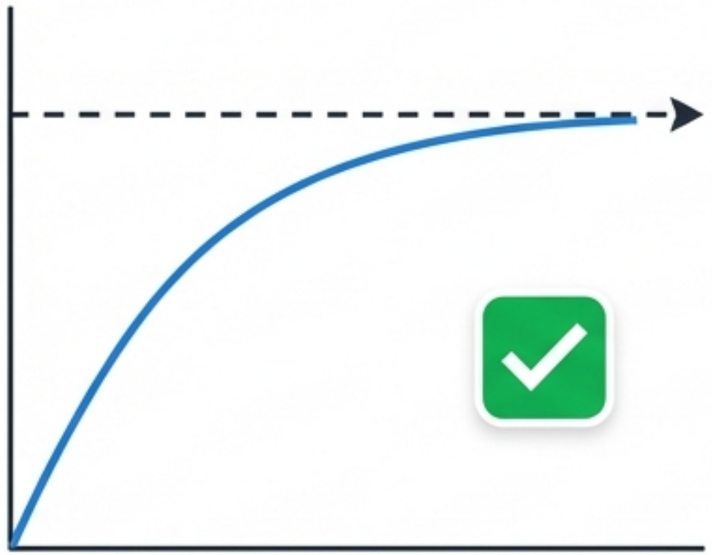
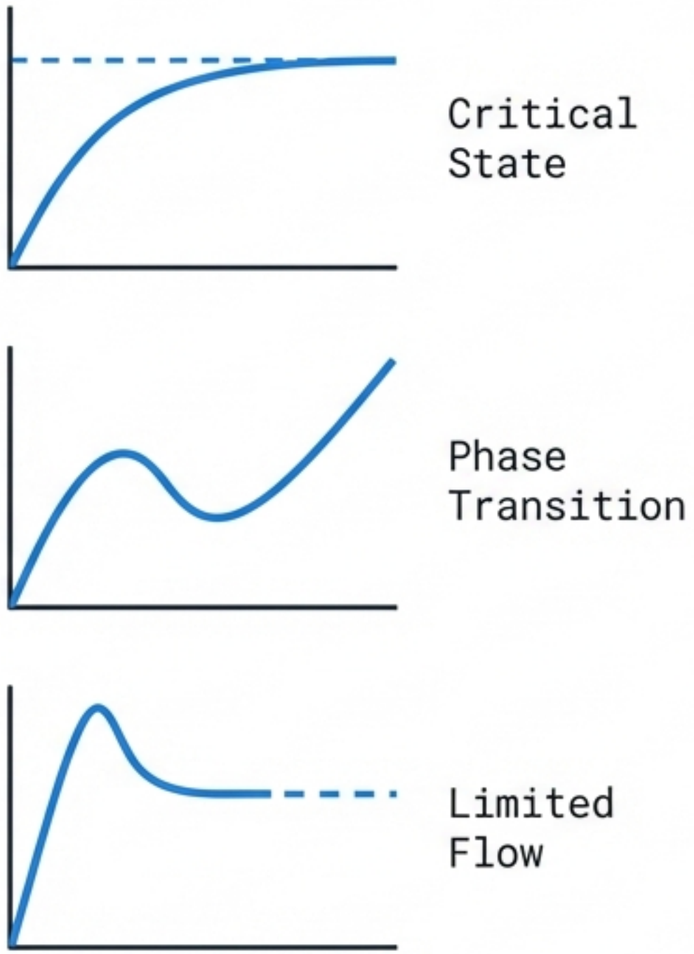
Evaluation Framework: Beyond R^2

Extrapolability

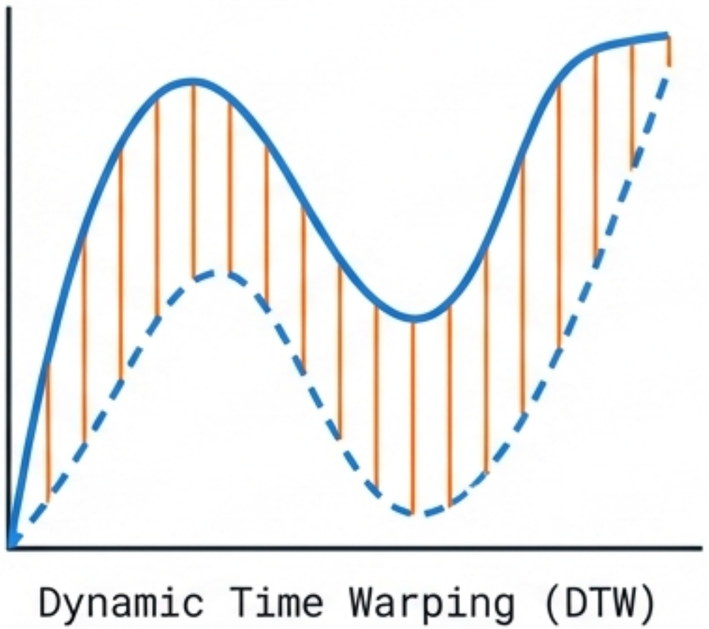


Does the FEM simulation converge or crash?

Physical Compliance



Fitting Ability



Measures shape similarity even with mismatched data resolution.

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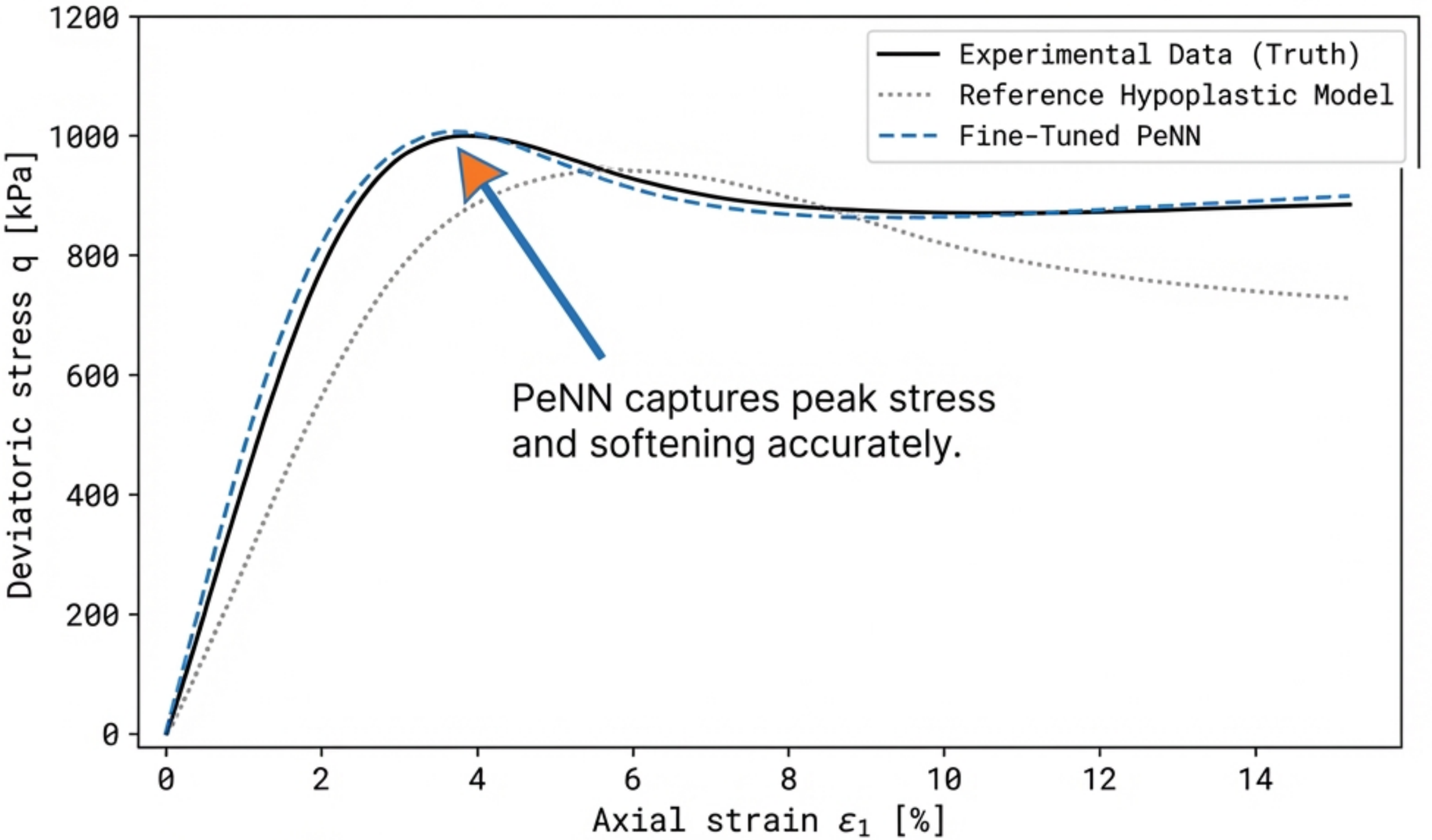
Results: Drained Triaxial Tests



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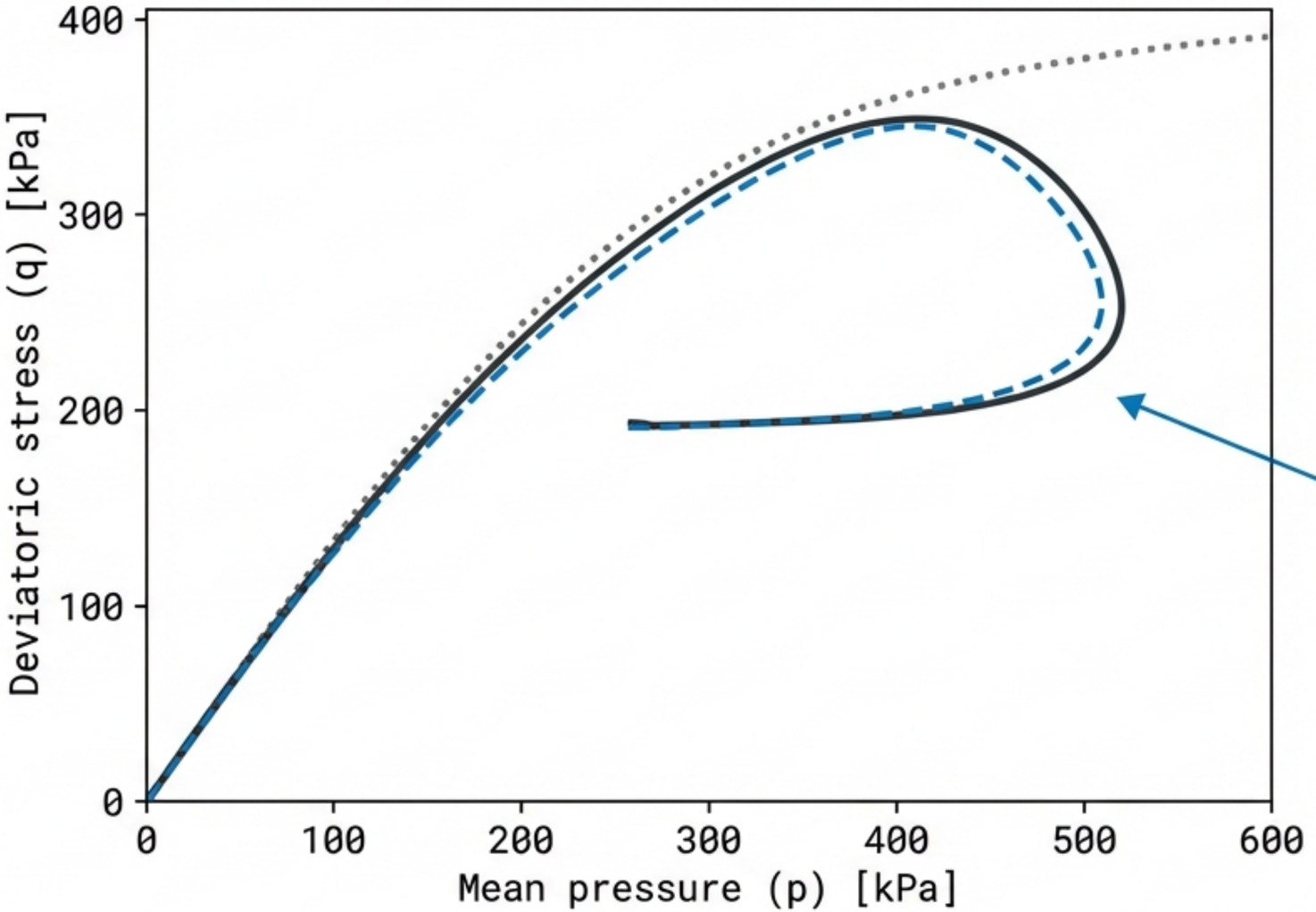
Technical Precision meets Swiss Editorial

Results: Undrained Triaxial Tests



Physics Challenge: Capturing "Limited Flow" instability. Without constraints (E_a), models often drift into negative pressure (unphysical).

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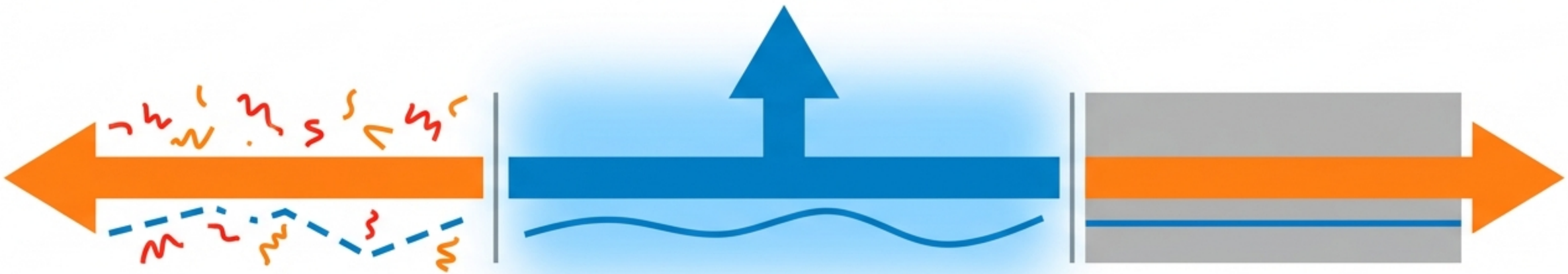
Optimizing the Transfer: What to Freeze?

Freeze 0 Layers

Optimum: Freeze First 2 Layers

Freeze All Layers



Overfitting.
Chases noise.
Unstable Physics.

Balanced.
Retains General Physics,
Learns Specific Reality.

Underfitting.
Too Rigid.
Fails to adapt.

Large Batch Size (2048) + Low Learning Rate = Filters experimental noise effectively.

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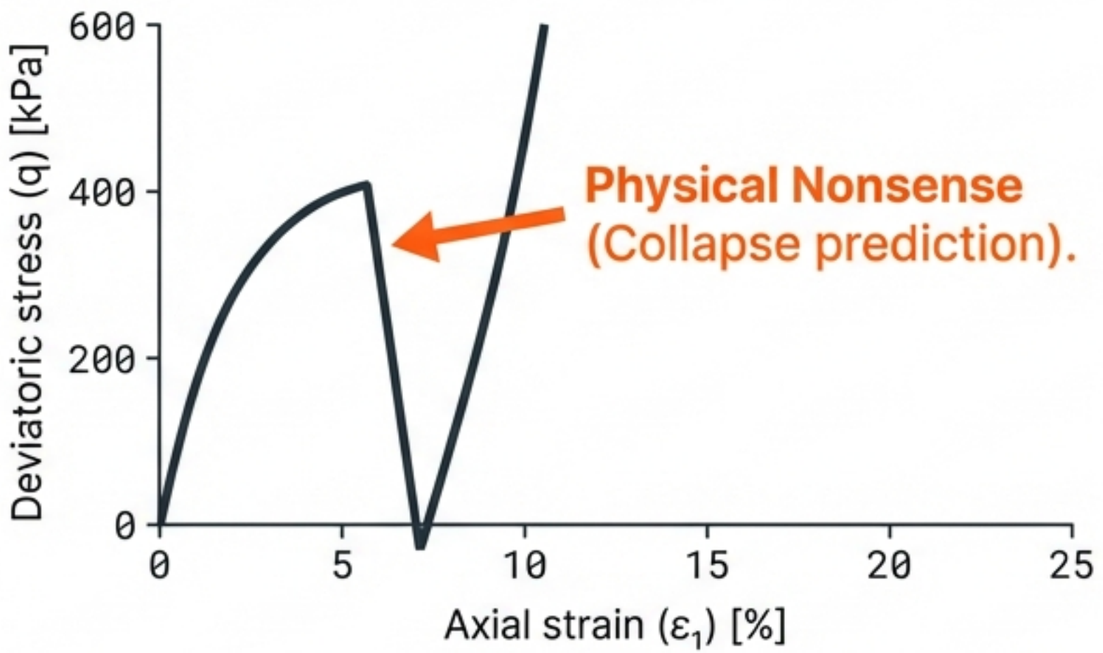
Addressing Extreme Scarcity: Synthetic Augmentation

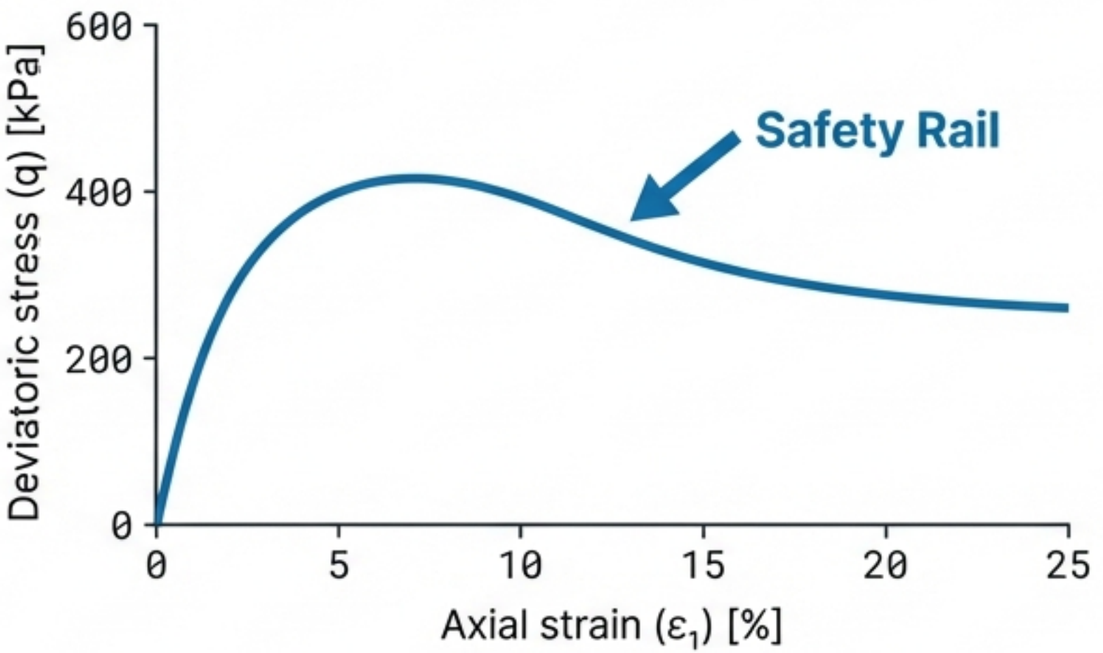
Scenario: 5 Drained Tests, 0 Undrained Tests

Without Augmentation



Model drifts into unphysical territory due to lack of data constraints.

With Synthetic Augmentation



Mixing real drained data with synthetic undrained data prevents unphysical predictions.

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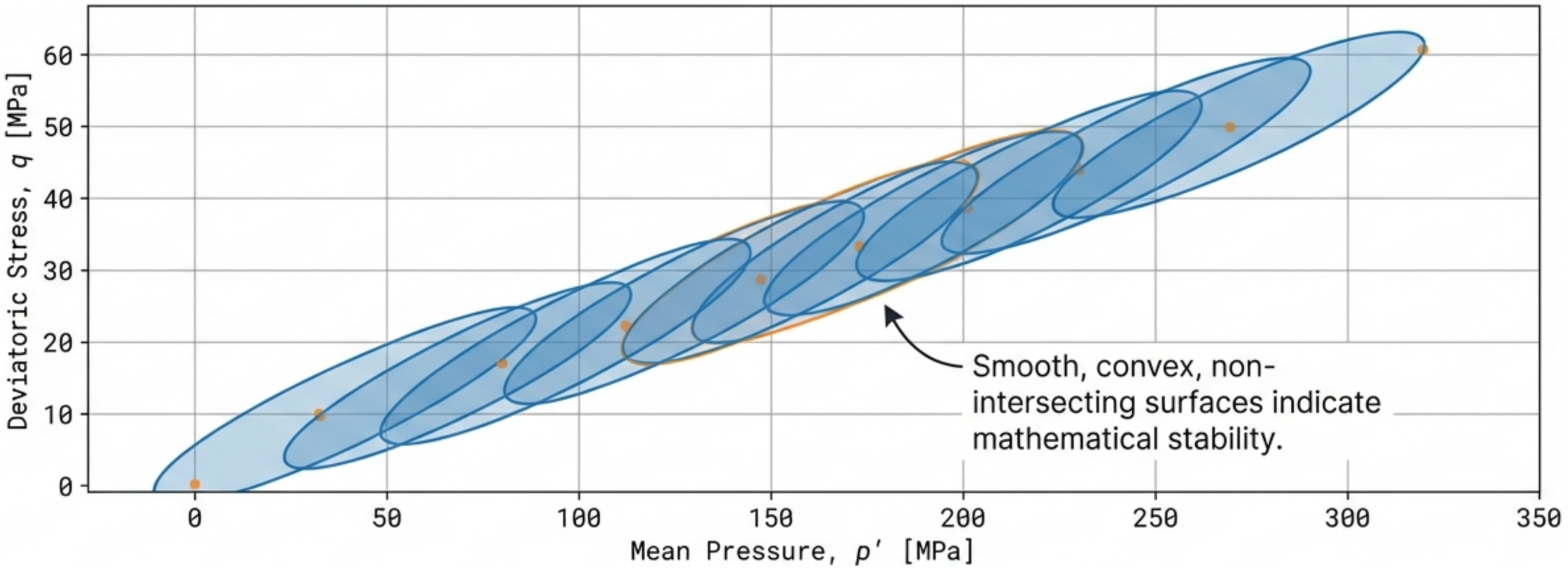
Verification: Model Response Envelopes



Proof that the data-driven model preserves convexity required for robust numerical solvers.

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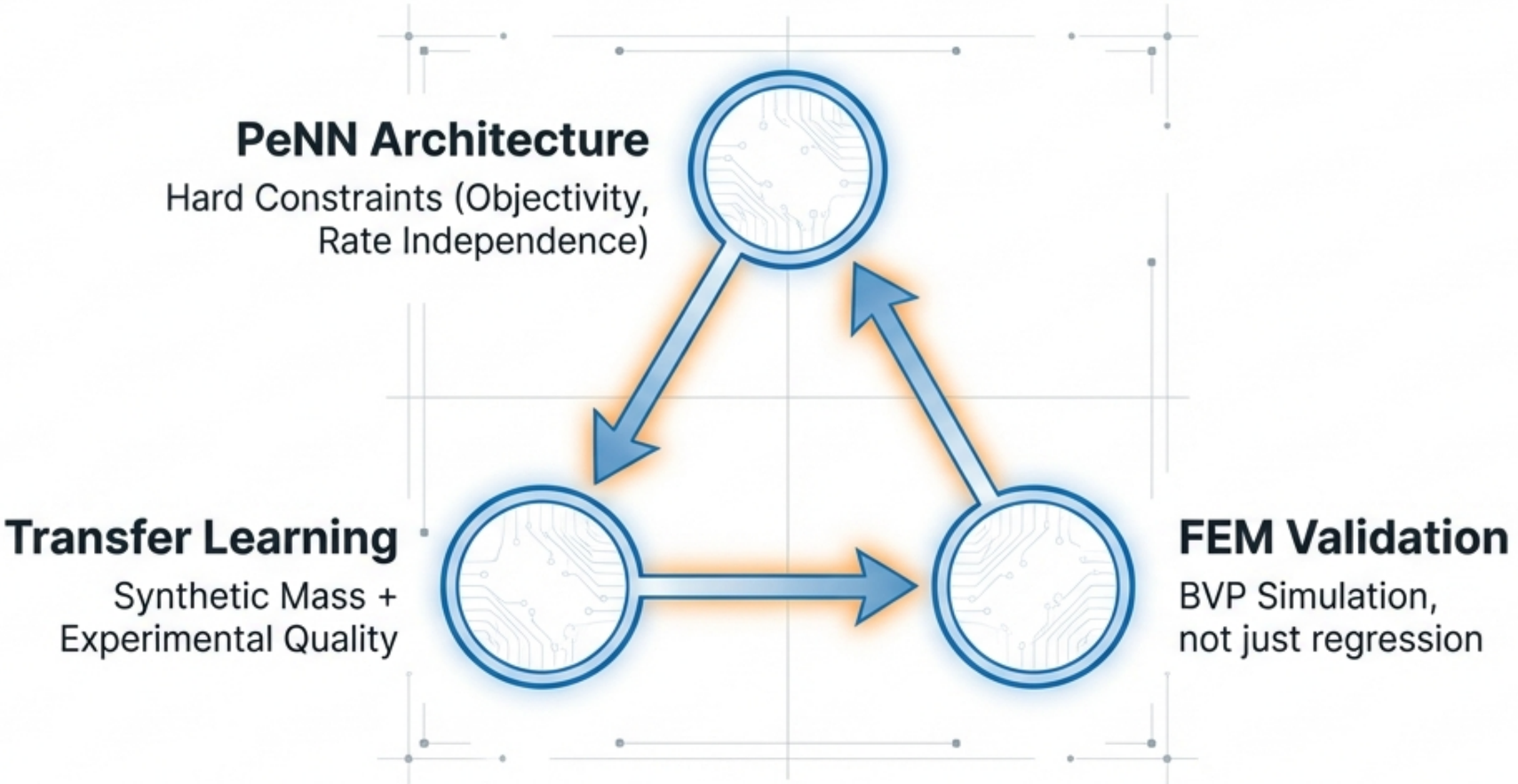
The Future of Data-Driven Constitutive Modeling



Hybrid AI-Physics workflows allow us to build superior engineering models using minimal, expensive experimental data

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